Why does $e^{i\pi}+1=0$?

Recall the Taylor and McLaurin series expansions for e^x, $\sin x$, $\cos x$:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots \qquad (1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} + \ldots \qquad (2)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^{n-1}\frac{x^{2n-2}}{(2n-2)!} + \ldots \qquad (3)$$

Let $x = i\theta$ in equation #1:

$$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \frac{x^4}{4!} + \ldots \qquad (4)$$

Let $x = \theta$ in equation #2, then multiply by i:

$$i\sin\theta = i\theta - \frac{i\theta^3}{3!} + \frac{i\theta^5}{5!} - \ldots \qquad (5)$$

Let $x = \theta$ in equation #3,

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots \qquad (6)$$

By adding equations #5 and #6, we get equation #4, so

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad (7)$$

(This is known as Euler's Identity)

Now substitute π for θ in equation #7:

$$e^{i\pi} = \cos\pi + i\sin\pi$$

$$e^{i\pi} = -1 + i(0)$$

Hence, $e^{i\pi} + 1 = 0$