$$
\text { Why does } e^{i \pi}+1=0 ?
$$

Recall the Taylor and McLaurin series expansions for $e^{x}, \sin x, \cos x$ :

$$
\begin{align*}
& e^{x}=1+x+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}+\ldots+\frac{x^{n}}{n!}+\ldots  \tag{1}\\
& \sin x=x-\frac{x^{3}}{3!}+\frac{x^{5}}{5!}-\ldots+(-1)^{n-1} \frac{x^{2 n-1}}{(2 n-1)!}+\ldots  \tag{2}\\
& \cos x=1-\frac{x^{2}}{2!}+\frac{x^{4}}{4!}-\ldots+(-1)^{n-1} \frac{x^{2 n-2}}{(2 n-2)!}+\ldots \tag{3}
\end{align*}
$$

Let $x=i \theta$ in equation \#1:

$$
\begin{equation*}
e^{i \theta}=1+i \theta-\frac{\theta^{2}}{2!}-i \frac{\theta^{3}}{3!}+\frac{x^{4}}{4!}+\ldots \tag{4}
\end{equation*}
$$

Let $x=\theta$ in equation $\# 2$, then multiply by $i$ :

$$
\begin{equation*}
i \sin \theta=i \theta-\frac{i \theta^{3}}{3!}+\frac{i \theta^{5}}{5!}-\ldots \tag{5}
\end{equation*}
$$

Let $x=\theta$ in equation \#3,

$$
\begin{equation*}
\cos \theta=1-\frac{\theta^{2}}{2!}+\frac{\theta^{4}}{4!}-\ldots \tag{6}
\end{equation*}
$$

By adding equations \#5 and \#6, we get equation \#4, so

$$
\begin{equation*}
e^{i \theta}=\cos \theta+i \sin \theta \tag{7}
\end{equation*}
$$

(This is known as Euler's Identity)

Now substitute $\pi$ for $\theta$ in equation \#7:

$$
\begin{aligned}
& e^{i \pi}=\cos \pi+i \sin \pi \\
& e^{i \pi}=-1+i(0)
\end{aligned}
$$

Hence, $\quad e^{i \pi}+1=0$

