Turvy with Applications of the Derivative -- A Puzzle by David Pleacher

Back in 1953, Roger Price invented a minor art form called the Droodle, which he described as "a Borkley-looking sort of drawing that doesn't make any sense until you know the correct title." In 1985, *Games* Magazine took the Droodle one step further and created the Turvy. Turvies have one explanation right-side-up and an entirely different one turned topsy-turvy. The Turvy above was created by John Ingelin of St. Bonifacius, Minnesota and published in *Games* Magazine in May 1986.

Here is the title right-side-up:

"__ __ __ __ __ __ __   __ __ __ __   __ __ __ __ __ __ __   __ __ __ __ __   __ __ __ __ __ ."

16 10 18 4 16 18 7   17 5 9 2   10 13 8 8 16 7 6   12 16 3 3 18   14 13 11 6 5

Here is the title upside-down:

"__ __ __ __ __ __ __   __ __   __   __ __ __ __ __ __ __   __ __ __ __ __   __ __ __ ."

17 4 13 8 9 11 12   13 2   18   17 18 15 15 18 6 9   12 18 10 17 5   1 16 14

To determine the titles to this turvy, solve the 18 application of derivatives problems on the next two pages. Then replace each numbered blank with the letter corresponding to the answer for that problem.

Derivative Application Problems:

1. Find the equation of the line normal to the curve $f(x) = x^3 - 3x^2$ at the point (1, -2).

2. Find the equation of the line tangent to the curve $x^2y - x = y^3 - 8$ at the point where x = 0.

3. Determine the point(s) of inflection of $f(x) = x^3 - 5x^2 + 3x + 6$.

4. Determine the relative minimum point(s) of $f(x) = x^4 - 4x^3$.

5. A particle moves along a line according to the law $s = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.
   Determine the total distance traveled between t = 0 and t = 4.

6. A particle moves along a line according to the law $s = t^4 - 4t^3$, where $t \geq 0$.
   Determine the total distance traveled between t = 0 and t = 4.

7. If one leg, AB, of a right triangle increases at the rate of 2 inches per second, while the other leg, AC, decreases at 3 inches per second, determine how fast the hypotenuse is changing (in **feet** per second) when AB = 6 feet and AC = 8 feet.

8. The diameter and height of a paper cup in the shape of a cone are both 4 inches, and water is leaking out at the rate of ½ cubic inch per second. Determine the rate (in inches per second) at which the water level is dropping when the diameter of the surface is 2 inches.

9. For what value of y is the tangent to the curve $y^2 - xy + 9 = 0$ vertical?

10. For what value of k is the line $y = 3x + k$ tangent to the curve $y = x^3$?

11. Determine the slopes of the two tangents that can be drawn from the point (3, 5) to the parabola $y = x^2$.

12. Determine the area of the largest rectangle that can be drawn with one side along the x-axis and two vertices on the curve $y = e^{-x^2}$.

13. A tangent drawn to the parabola $y = 4 - x^2$ at the point (1, 3) forms a right triangle with the coordinate axes. What is the area of this triangle?

14. If the cylinder of largest possible volume is inscribed in a given sphere, determine the ratio of the volume of the sphere to that of the cylinder.

15. Determine the first quadrant point on the curve $y^2x = 18$ which is closest to the point (2, 0).

16. Two cars are traveling along perpendicular roads, car A at 40 mph, car B at 60 mph. At noon when car A reaches the intersection, car B is 90 miles away, and moving toward it. At 1PM, what is the rate, in miles per hour, at which the distance between the cars is changing?

17. A 26-foot ladder leans against a building so that its foot moves away from the building at the rate of 3 feet per second. When the foot of the ladder is 10 feet from the building, at what rate is the top moving down (in feet per second)?

18. A rectangle of perimeter 18 inches is rotated about one of its sides to generate a right circular cylinder. What is the area, in square inches, of the rectangle that generates the cylinder of largest volume?

Answers:

A. 18	B. $\left(3,\sqrt{6}\right)$	C. $\frac{5}{4}$	D. $\sqrt{3}:1$	E. ± 3
F. 12y + x = 24	G. 54	H. 34	I. -4	J. $\pm\sqrt{3}$
K. $3y - x = -7$	L. $\left(3,-27\right)$	M. ± 1	N. $-\frac{1}{10}$	O. $\frac{25}{4}$
P. $\sqrt{\frac{2}{e}}$	Q. $\sqrt{3}:3$	R. $\sqrt{2e}$	S. $\frac{1}{2\pi}$	T. ± 2
U. 2 and 10	V. $\frac{2}{e}$	W. $\left(6,\sqrt{3}\right)$	X. $\frac{1}{5}$	Y. $2\sqrt{3}:3$
Z. $\left(\frac{5}{3},\frac{47}{27}\right)$				

Many Thanks to Samuel Iofel for correcting answer N.