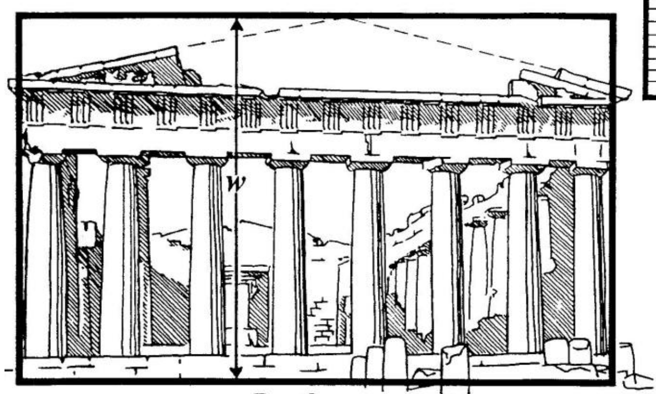
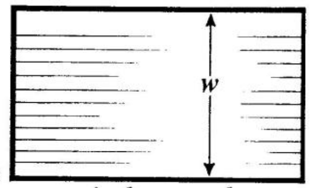
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
STUDENT math notes
SEPTEMBER 1986

Golden Rectangles and Ratios

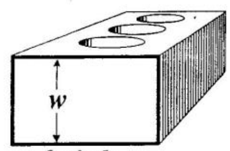
All the figures below have a rectangular shape. What else do they have in common? This particular rectangular shape, called the *golden rectangle,* is considered to be the most pleasing to the eye.



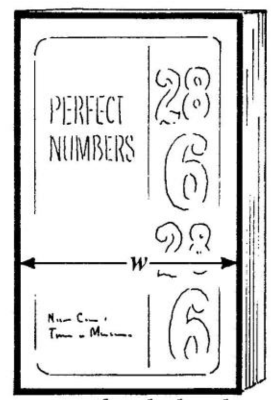
Parthenon



index card



brick



paperback book

Measure the length and width of each rectangle in millimeters. Express the ratio of the width to the length as a decimal rounded to two places.

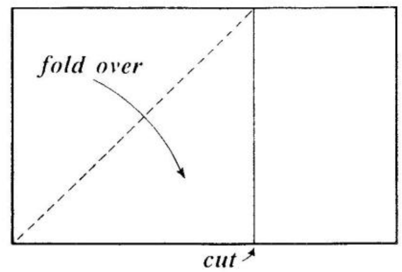
	Width (w)	Length (l)	w/l	Decimal Ratio
Parthenon	_____	_____	_____	_____
Index card	_____	_____	_____	_____
Paperback book	_____	_____	_____	_____
Brick	_____	_____	_____	_____

By expressing these ratios in decimal form, we can observe that each of them is approximately 0.6. The ratio 0.61803 . . . is called the *golden ratio*.

The editors wish to thank **Rick Billstein** and **Johnny W. Lott,** University of Montana, Missoula, MT 59812, for writing this issue of the *NCTM Student Math Notes.*

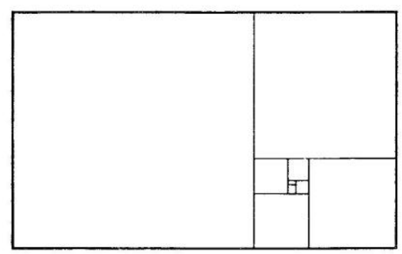
A Physical Model for Generating Golden Rectangles

Cut a sheet of paper to measure 25 cm by 15.5 cm. This rectangle closely approximates a golden rectangle. Fold over one corner of the rectangle as shown in the adjacent figure. Then cut off the square from the rectangle. The remaining rectangle has the same proportions as the original rectangle; hence it is also a golden rectangle.



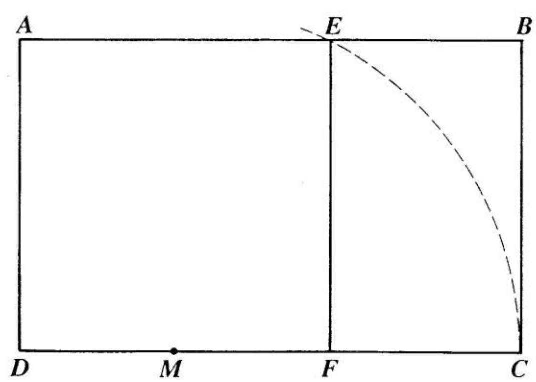
You can continue to generate golden rectangles by repeating this process. Each successive square has sides approximately 0.61803 . . . times the length of the sides of the preceding square.

Place the squares together to form the original golden rectangle as in the figure shown here. This representation of the golden rectangle is often referred to as "the rectangle with the whirling squares."



Constructing a Golden Rectangle

A golden rectangle can be constructed using a compass and straightedge. To accomplish this construction, follow the steps given below. The figure shows the appropriate lettering of the vertices.



1. Construct a square *AEFD*.
2. Bisect $\overline{DF}$. Label the midpoint *M*.
3. Extend $\overline{DF}$.
4. With center *M* and radius *ME*, draw an arc intersecting $\overleftrightarrow{DF}$ at *C*.
5. Construct a perpendicular to $\overline{DC}$ at *C*.
6. Extend $\overline{AE}$ to intersect the perpendicular at *B*.

The rectangle *ABCD* is a golden rectangle. It can be shown that *BCFE* is also a golden rectangle. To show that this statement is true, assume $MF = 1$ and find each of the following lengths:

(*a*) *FE* ______ (*b*) *BC* ______ (*c*) *ME* ______ (*d*) *MC* ______ (*e*) *FC* ______ (*f*) *DC* ______

Find the decimal values of these ratios: *BC*/*DC* ______ *FC*/*BC* ______ Are they equivalent?