# Slot Car Racing – An application of Circumference

by Eugene Krauss, University of Michigan in the May 1992 Mathematics Teacher

Slot car racing is an application that requires no advanced mathematics -- just the formula for the circumference of a circle. Youngsters are familiar with the small electrically powered cars that race along parallel slots in track sections that can be assembled in a variety of ways. The figures below show some possible layouts. Notice that the curved track sections can be joined to form quarter-circles and semicircles of different radii and that overpasses/underpasses are possible.

**Question #1:** In the simple track layout shown below (figure 1), what head start should the car in the dotted slot be allowed? The slots are 4 centimeters apart.

**Solution:** Straightaway sections can be ignored. We can imagine a solid circle of radius $R$ centimeters and a concentric dotted one of radius $(R + 4)$ centimeters. The difference in their circumference is as follows:
$2\pi (R + 4) - 2\pi R = 8\pi$ cm ~ 25 cm.
The car in the dotted slot should be given a 25-centimeter head start. Or putting it differently, if equally fast cars begin at the same starting line, then the inside car should win a one lap race by $8\pi$ centimeters. Emphasize that this winning margin is independent of how sharp or gradual the semicircular ends of the track happen to be.

**Question #2:** Suppose the track is laid out as in figure 2, and suppose two equally fast cars start off together from the starting line. Which should win a one-lap race -- the dotted or the solid -- and by how much?

**Solution:** We have seen that for equally fast cars on a circular track, the inside car wins by $8\pi$ centimeters; so on a semicircular curve the inside car gains $4\pi$ centimeters and on a quarter-circular curve it picks up $2\pi$ centimeters. We can chart the progress of the race as follows:

| **After completing turn** | **Dotted car ahead by** |
|---|---|
| **I** | **$4\pi$** |
| **II** | **$6\pi$** |
| **III** | **$2\pi$** |
| **IV** | **0** |

The race should end in a tie. In other words, this is a fair track. No handicap needs to be given to either car. The solid path and the dotted path have the same length. Note: for this question, the distance between slots, 4 centimeters, turned out to be irrelevant, but it made the argument a bit more concrete.

**Question #3:** Determine by how much the dotted car should be expected to win a one-lap race on each of the layouts in figure 3.

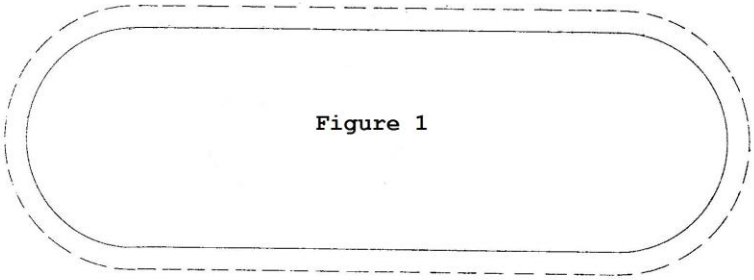


Figure 1

SLOT CAR RACING

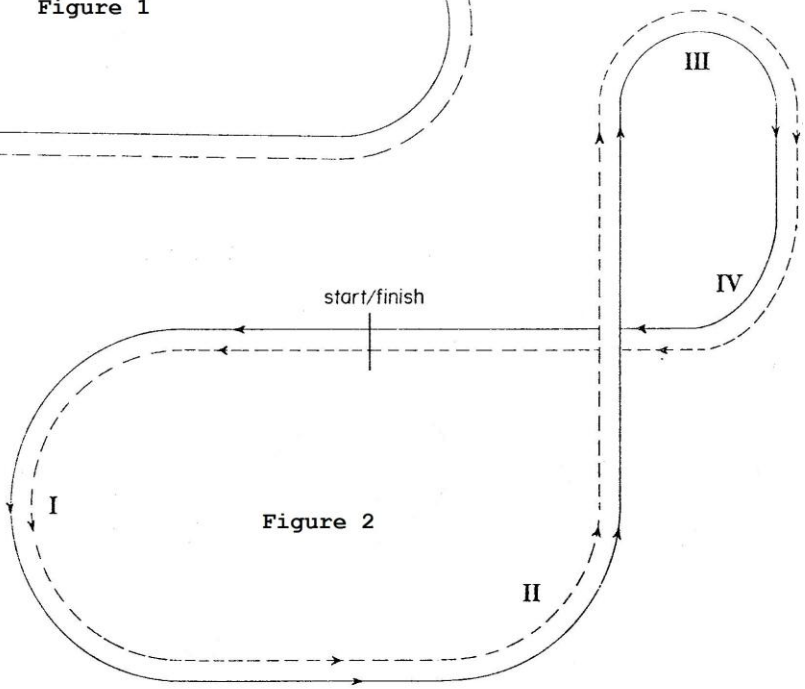
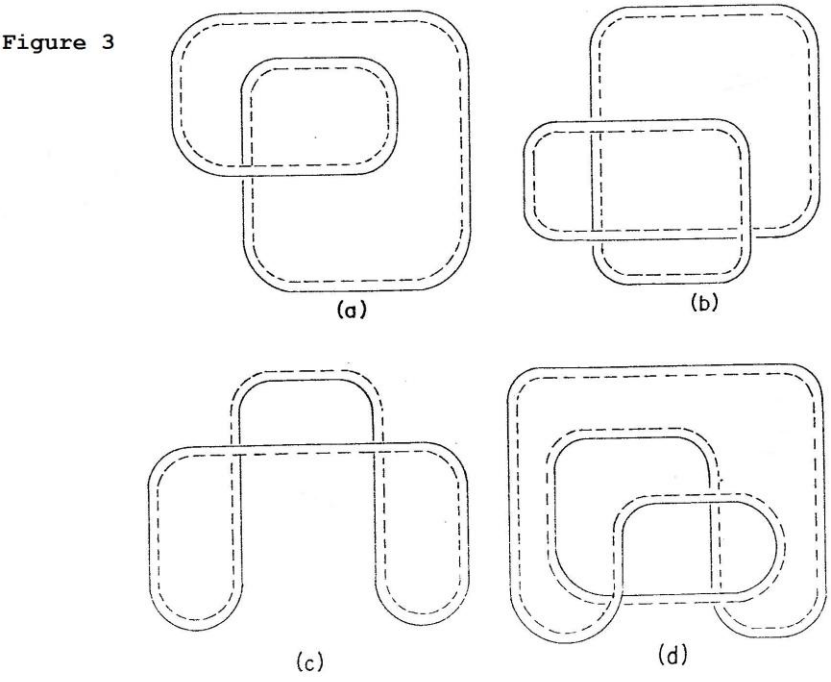



Figure 2

Figure 3



Most slot car race tracks have crossover pieces to help make the races fair.

**Put in crossover pieces to make the following tracks fair:**