March Madness - Math Style

Thoughts from David Pleacher
With the NCAA brackets announced and March Madness officially upon us (and spilling into April), news broadcasts (not just sports reports) are full of advice on how people should fill out their brackets.
One of these reports featured a math professor from the University of Miami.
He claimed that there are 9 quintillion possible ways to predict the outcome of the NCAA tournament. That's a pretty huge number.
Also, he said that number is "comparable to the grains of sand on all the beaches in the world." That comparison seemed like a stretch.

Well, here is an explanation.
For the bracket, there are only 63 games (they don't count the four play-in games).
Each game has two possible outcomes.
In two games, there would be $2 \times 2=4$ possible outcomes.
If Team A and Team B play in the first game and Team C and Team D play in the second game, here are the possible outcomes (winners):
A-C A-D B-C B-D
For three games, there are $2 \times 2 \times 2=8$ possible outcomes.
Using Teams A and B. C and D, and E and F, here are the outcomes:
A-C-E A-C-F A-D-E A-D-F B-C-E B-C-F B-D-E B-D-F
So, in the 63 games in the bracket, there would be
2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x
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Here is the answer:
$2^{\wedge} 63=9,223,372,036,854,775,808$
So, the Miami professor was right -- it is over 9 quintillion ways. As far as the sand metaphor goes, I would guess that it is true. My mind can't even fathom what a quintillion looks like. But here is another way to look at 9 quintillion.

As of 2022, there were 8.1 billion people in the world. In order to cover all 9 quintillion possible brackets, each person on earth would have to fill out $1,138,687,905$ brackets. That's over one billion brackets per person.
If each person could fill out a bracket in one minute, it would take over 790,755 days or more than 2,166 years to complete!

Here are the odds of correctly predicting March Madness Games at various stages of the bracket:

| \# of Games | A Perfect ... | Odds |  |
| :---: | :--- | :--- | :--- |
| 32 | first round | 1 in 4.29 billion | $2^{\wedge} 32=4,294.967,296$ |
| 48 | Sweet Sixteen | 1 in 281.4 trillion | $2^{\wedge} 48=2.814749767 \times 10^{14}$ |
| 56 | Elite Eight | 1 in 72.1 quadrillion | $2^{\wedge} 56=7.205759404 \times 10^{16}$ |
| 60 | Final Four | 1 in 1.15 quintillion | $2^{\wedge} 60=1.152921505 \times 10^{18}$ |
| 62 | Semi Finals | 1 in 4.61 quintillion | $2^{\wedge} 62=4.611686018 \times 10^{18}$ |
| 63 | Complete Bracket |  |  |

If you included the four play-in games in the bracket so that there were 68 teams and 67 games, then these are the odds of correctly picking the various stages of the bracket:

| \# of Games | A Perfect... | Odds |
| :---: | :---: | :---: |
| 36 | first round | 1 in 68.7 billion |
| 52 | Sweet Sixteen | 1 in 4.5 quadrillion |
| 60 | Elite Eight | 1 in 1.2 quintillion |
| 64 | Final Four | 1 in 18.4 quintillion |
| 66 | Championship Game | 1 in 73.8 quintillion |
| 67 | bracket | 1 in 147.6 quintillion |

