Coloring Maps

Mapmakers follow two rules in coloring maps:

- Each country or state should be colored with a single color.
- Different colors should be used for countries or states that share a common border.

Figure 1 is a map of Utah, Colorado, Arizona, and New Mexico. We could use a different color for each state, but following the mapmakers' rules, we can use fewer colors. Color the map with the fewest possible colors. If the states touch at only a single point, they can be of the same color.

1. How many colors are required?_____

Fig. 1

Fig. 2

Figure 2 is a map of another portion of the western United States. How many colors do we need for this map if we follow the mapmakers' rules?

2. Are two colors enough?_____

3. If not, can the map be colored with three colors?

_____

4. What is the minimum number of colors required to color the map that combines figures 1 and 2?

_____

Coloring Maps—Continued

Test your conjecture by coloring figure 3.

Fig. 3

For each of the regional maps in figure 4, determine the fewest colors necessary to color the states following the mapmakers' rules.

5. Which maps require only three colors?_____

6. Which maps require four colors?_____

7. Do any require five colors?_____

8. Is it possible for a map to require five colors?_____
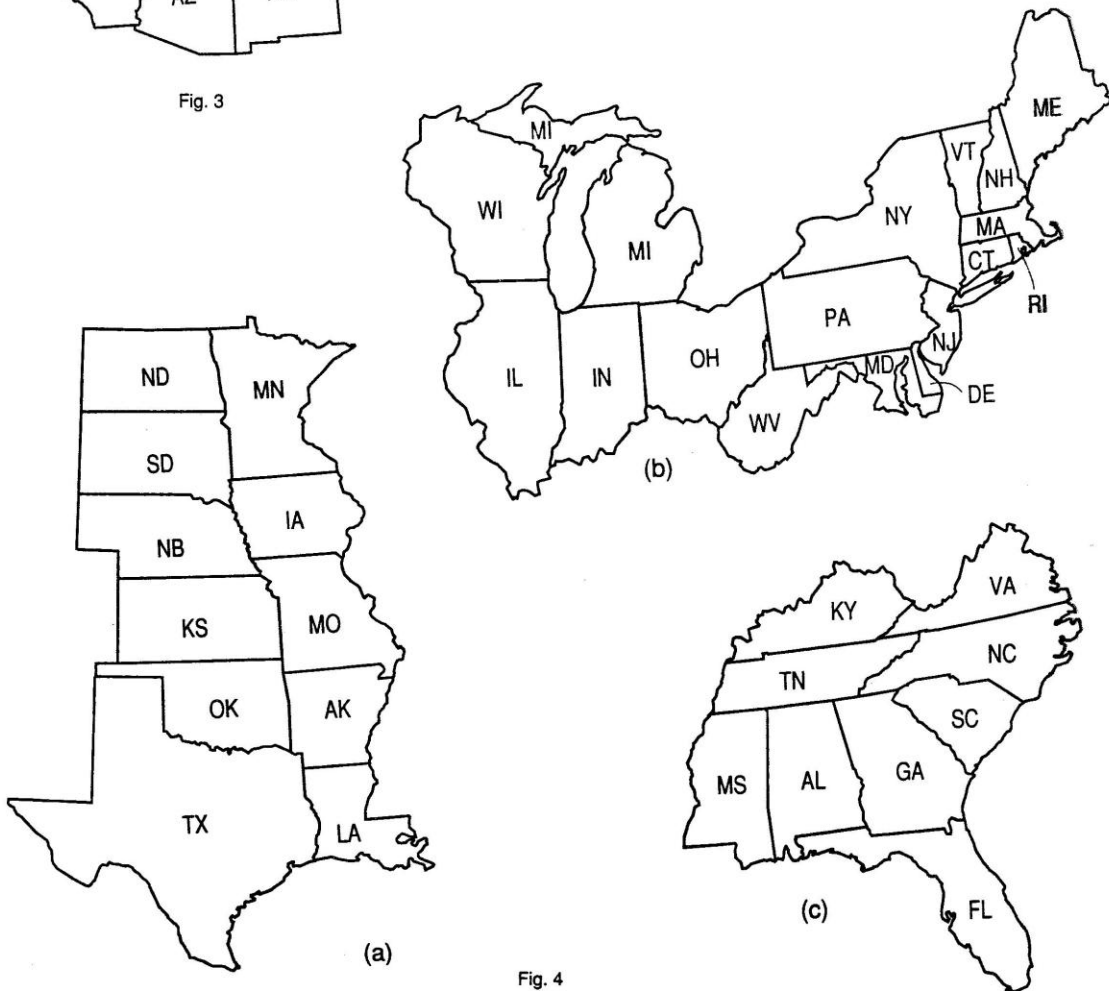

(a)

(b)

(c)

Fig. 4

Coloring Maps—Continued

9. Consider figure 5, in which country A is separated into two pieces. Are five colors necessary?_____

10. Are any states in the United States broken into more than one piece?_____Which ones?_____

11. Is it possible to create a map that would require six colors?_____

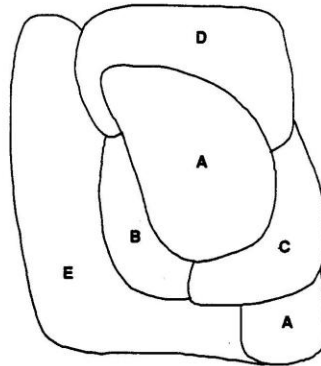12. Is it possible to create a map requiring five colors in which none of the countries are broken?_____

Fig. 5

Figure 6 shows a map of the continental United States. Four colors are enough to color this map. Color it using the mapmakers' rules.
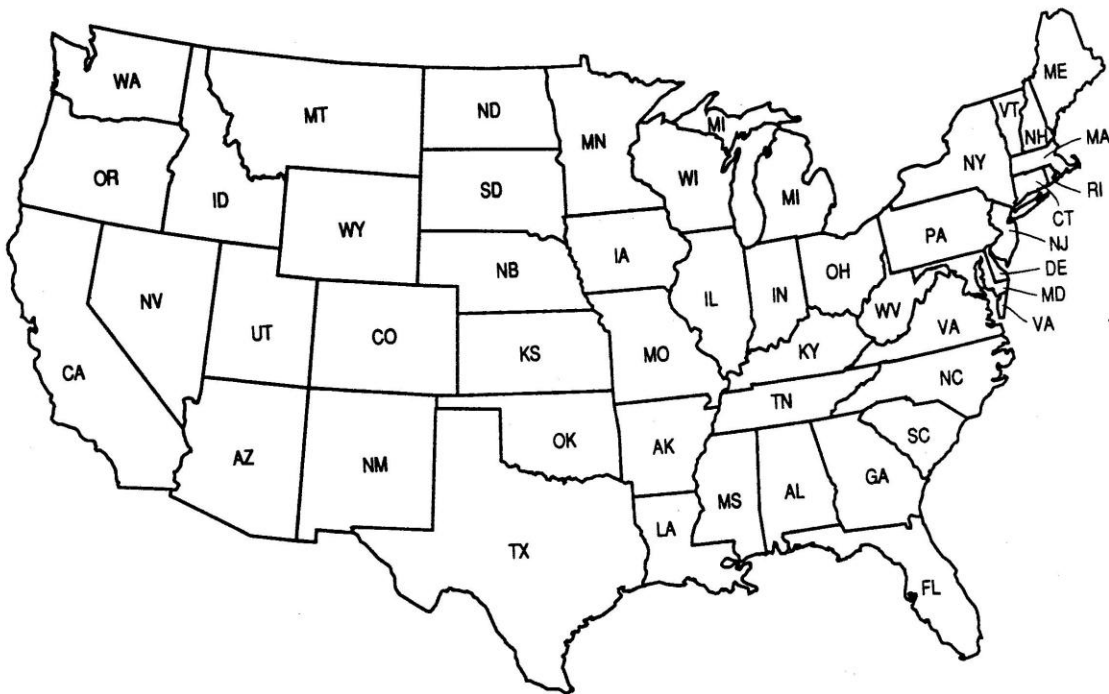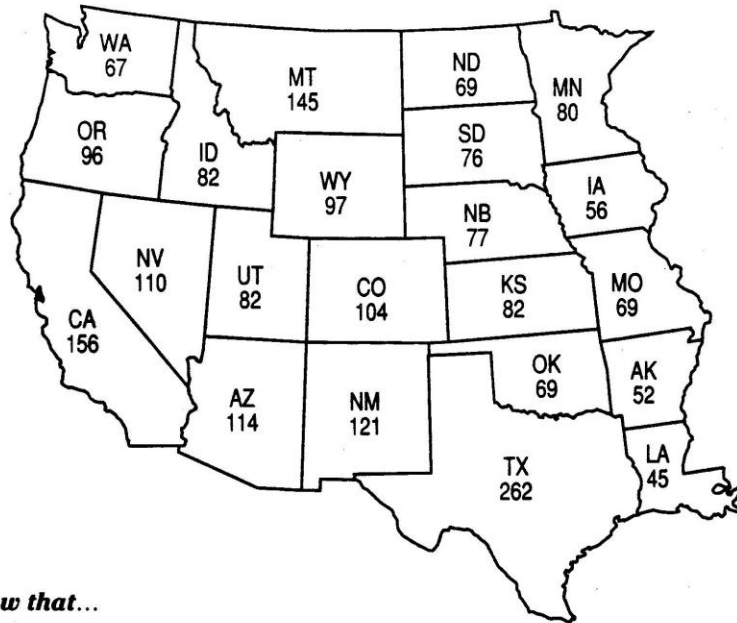

Fig. 6

Can you...

• color the map below at a minumum cost if the four chosen colors cost $1, $2, $3, and $4 per 1000 square miles, respectively? The number in each state represents thousands of square miles.

Did you know that...

• if a map is drawn on a flat plane or a globe and if all the countries or states are single, unbroken regions, then four colors always suffice to color the map (four-color theorem)?

• the four-color theorem was verified in 1976 at the University of Illinois with the help of computers?

• if we draw our map on a doughnut, the map may require as many as seven different colors?

Bibliography

• "Thinking with Ink." In Problem-solving Strategies. (Computer program) Available from Minnesota Educational Computing Corporation, 3490 Lexington Avenue North, St. Paul, MN 55126. Challenges students to color maps with inks of varying costs.

Answers:

1. Two	4. Four	7. No	10. Yes; Michigan and Virginia
2. No	5. a, b, c	8. Maybe	11. Yes
3. Yes	6. None	9. Yes	12. Not if the map is in a plane