## A Droodle for the S.A.T. Math Exam with a calculator

 A puzzle by David Pleacher
"A Droodle is a borkley looking sort of drawing that doesn't make any sense until you know the correct title." - Roger Price

Captions for the picture:

|  | T | O | M | A | T | $\mathbf{O}$ | S | A | $\mathbf{N}$ | D | W | $\mathbf{I}$ | $\mathbf{C}$ | $\mathbf{H}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\overline{20}$ | $\overline{18}$ | $\overline{16}$ | $\overline{19}$ | $\overline{20}$ | $\overline{25}$ |  | $\overline{14}$ | $\overline{19}$ | $\overline{15}$ | $\overline{1}$ | $\overline{10}$ | $\overline{24}$ | $\overline{23}$ |.



|  | H | O | T | D | O | G | O N | S | K I | S |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\overline{4}$ | $\overline{18}$ | $\overline{20}$ | $\overline{1}$ | $\overline{25}$ | $\overline{6}$ | $\overline{18}$ | $\overline{15}$ | $\overline{21}$ | $\overline{3}$ | $\overline{11}$ |
| 14 |  |  |  |  |  |  |  |  |  |  |  |.


|  | T | $\mathbf{R}$ | $\mathbf{Y}$ | $\mathbf{I}$ | $\mathbf{N}$ | $\mathbf{G}$ | T | $\mathbf{O}$ | P | $\mathbf{I}$ | $\mathbf{C}$ | $\mathbf{K}$ | U | $\mathbf{P}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\overline{20}$ | $\overline{22}$ | $\overline{5}$ | $\overline{24}$ | $\overline{15}$ | $\overline{6}$ | $\overline{20}$ | $\overline{25}$ | $\overline{2}$ | $\overline{24}$ | $\overline{23}$ | $\overline{3}$ | $\overline{8}$ | $\overline{2}$ |  |
| 19 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |


| C | $\mathbf{H}$ | $\mathbf{E}$ | $\mathbf{R}$ | $\mathbf{R}$ | $\mathbf{Y}$ | W | $\mathbf{I}$ | T | $\mathbf{H}$ |  | C | $\mathbf{H}$ | O | P | S | T | I | C | K |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\overline{23}$ | 4 | 17 | 13 | 22 | $\overline{5}$ | $\overline{10}$ | $\overline{11}$ | $\overline{20}$ | $\overline{4}$ |  | $\overline{23}$ | $\overline{4}$ | $\overline{25}$ | $\overline{2}$ | $\overline{21}$ | $\overline{20}$ | $\overline{24}$ | $\overline{23}$ | $\overline{12}$ |
|  | $\overline{21}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |.

D 1. If $y=k x$, where $k$ is a constant, and $y=24$ when $x=6$, what is the value of $y$ when $x=5$ ?
First solve for $k$ to get 4. Then, when $x=5, y=4(5)=20$

P 2. In the figure below, lines $l$ and $m$ are parallel and lines $s$ and $t$ are parallel. If the $m \angle 1=35^{\circ}$, what is the measure of $\angle 2$ ?


Use corresponding angles twice to get the supplement of $\angle 2$ to be 35 degrees. So, the measure of $\angle 2=145^{\circ}$.
$\qquad$ 3. The table below shows some values of the linear function $f$.

| $n$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $f(n)$ | -2 | 1 | 4 | 7 |

Which of the following defines $f$ ?

Since $f$ is linear, you can use any two points to find the equation.
Using, $(2,1)$ and $(3,4)$, you get the slope to be $m=3$.
Then $y-1=3(x-2)$, so $\quad f(n)=3 n-5$
$\qquad$ 4. At Rocky Mountain High School, approximately 7 percent of enrolled juniors and 5 percent of enrolled seniors were inducted into the National Honor Society last year. If there were 562 juniors and 602 seniors enrolled at Rocky Mountain High School last year, which of the following is closest to the total number of juniors and seniors at Rocky Mountain High School last year who were inducted into the National Honor Society?

Total $=.07 \times 562+.05 \times 602=69.44$, so the closest answer is H. 69

Y 5. If $\frac{3}{5} w=\frac{4}{3}$, what is the value of $w$ ?

Multiply by $\frac{5}{3}$ to get $\left(\frac{5}{3}\right) \frac{3}{5} w=\frac{4}{3}\left(\frac{5}{3}\right) \Rightarrow w=\frac{20}{9}$
$\qquad$ 6. Barbara walks 25 meters in 13.7 seconds. If she walks at this same rate, which of the following is closest to the distance she will walk in 4 minutes?

First determine Barbara's rate per second: 1.8248 meters per second.
Then multiply by 60 to get the number of meters she walked in 1 minute and then multiply by 4 to get the number pf meters walked in 4 minutes: 437.956 .
the closest answer is G. 450 meters.

B 7. If the function $f$ has five distinct zeros, which of the following could represent the complete graph of $f$ in the $x y$-plane?

Only the graph in B crosses the x-axis in exactly 5 places. The others cross in 4 places (C and U) or 6 places (E).
C.
B

E.

$\qquad$ 8. The equation $h=-16 t^{2}+v t+k$ gives the height $h$, in feet, of a ball $t$ seconds after it is thrown straight up with an initial speed of $v$ feet per second from a height of $k$ feet. Which of the following gives $v$ in terms of $h, t$, and $k$ ?

First isolate $v t$ on one side:

$$
v t=h+16 t^{2}-k
$$

Then divide each side by $t$ :

$$
v=\frac{h-k}{t}+16 t
$$

L 9. To make a bakery's signature chocolate muffins, a baker needs 2.5 ounces of chocolate for each muffin. How many pounds of chocolate are needed to make 48 signature chocolate muffins? (1 pound equals 16 ounces.)

Multiply $48 \times 2.5=120$ ounces.
Then divide by 16 to get the number of pounds: 7.5

W 10. The weight of an object on Venus is approximately $\frac{9}{10}$ of its weight on Earth. The weight of an object on Jupiter is approximately $\frac{23}{10}$ of its weight on Earth. If an object weighs 100 pounds on Earth, approximately how many more pounds does it weigh on Jupiter than it weighs on Venus?

Multiply the two fractions by 100 to get its weight on Venus and Jupiter. Then subtract $230-90=140$ pounds.
_ 11. An online bookstore sells novels and magazines. Each novel sells for $\$ 4$, and each magazine sells for $\$ 1$. If Audrey purchased a total of 11 novels and magazines that have a combined selling price of $\$ 20$, how many novels did she purchase?

Let $n=$ number of novels purchased and let $m=$ number of magazines purchased.

$$
\begin{aligned}
& n+m=11 \\
& 4 n+1 m=20 \\
& m=11-n, \quad 4 n+(11-n)=20 \\
& 3 n=9 \Rightarrow n=3
\end{aligned}
$$

K 12. The Downtown Business Association (DBA) in Fort Collins plans to increase its membership by a total of $n$ businesses per year. There were $b$ businesses in the DBA at the beginning of this year. Which function best models the total number of businesses, $y$, the DBA plans to have as members $x$ years from now?

$$
\text { K. } \quad y=n x+b
$$

$\underline{R}$ 13. Which of the following is an equivalent form of $(1.5 x-2.4)^{2}-\left(5.2 x^{2}-6.4\right)$ ?

$$
\begin{aligned}
& (1.5 x-2.4)^{2}-\left(5.2 x^{2}-6.4\right) \\
& 2.25 x^{2}-2(1.5 x)(2.4)+5.76-5.2 x^{2}+6.4 \\
& 2.25 x^{2}-7.2 x+5.76-5.2 x^{2}+6.4 \\
& -2.95 x^{2}-7.2 x+12.16
\end{aligned}
$$

S 14. The graph of which of the following equations is perpendicular to the graph of $-2 x+3 y=6$ ?

The slope of the given graph is $2 / 3$, so the slope of a line perpendicular must be $-3 / 2$.
SO, the answer is $3 x+2 y=6$

N 15. Determine the value of $x$ in the solution of the system of equations

$$
\begin{aligned}
& \frac{1}{2} y=4 \\
x- & \frac{1}{2} y=2 \\
x= & \frac{1}{2} y+2, \quad \text { so } \quad x=4+2=6
\end{aligned}
$$

M 16. Which of the following ordered pairs satisfies the following system of inequalities?

$$
\begin{aligned}
& y \leq 3 x+1 \\
& x-y>1
\end{aligned}
$$

Simple substitution shows that the only coordinates that satisfy both inequalities is:
M. $(2,-1)$

E 17. A polling agency recently surveyed 1,000 adults who were selected at random from a large city and asked each of the adults, "Are you satisfied with the quality of air in the city?" Of those surveyed, 78 percent responded that they were satisfied with the quality of air in the city. Based on the results of the survey, which of the following statements must be true?

1. Of all adults in the city, 78 percent are satisfied with the quality of air in the city.
2. If another 1,000 adults selected at random from the city were surveyed, 78 percent of them would report they are satisfied with the quality of air in the city.
3. If 1,000 adults selected at random from a different city were surveyed, 78 percent of them would report they are satisfied with the quality of air in the city.
E. None
G. 2 only
A. 1 and 2 only
D. 1 and 3 only

Choice A is correct. Statement 1 need not be true. The fact that $78 \%$ of the 1,000 adults who were surveyed responded that they were satisfied with the air quality in the city does not mean that the exact same percentage of all adults in the city will be satisfied with the air quality in the city. Statement 2 need not be true because random samples, even when they are of the same size, are not necessarily identical with regard to percentages of people in them who have a certain opinion. Statement 3 need not be true for the same reason that statement 2 need not be true: results from different samples can vary. The variation may be even bigger for this sample since it would be selected from a different city. Therefore, none of the statements must be true.

O 18. In $\triangle A B C$ below, what is the length of $\overline{A D}$ ?

$\triangle A B D \cong \triangle C B D$
$\therefore A B=B C=12$
Since $\overline{A D}$ is opposite a thirty degree angle, it must be half the hypotenuse,
So, the length of $\overline{A D}=6$ units.

A 19. In the equation $\frac{a-b}{a}=c$, if $a$ is negative and $b$ is positive, which of the following must be true?

The equation can be rewritten as $1-\frac{b}{a}=c$ or equivalently as $1-c=\frac{b}{a}$.

Since $a<0$ and $b>0$, it follows that
$\frac{b}{a}<0$, and so, $1-c<0$, or equivalently, $c>1$.

T 20. In Colorado, Mr. Harding's eighth-grade class consisting of 26 students was surveyed and 34.6 percent of the students reported that they had at least two siblings. The average eighth-grade class size in the state is 26 . If the students in Mr. Harding's class are representative of students in the state's eighth-grade classes and there are 1,800 eighth-grade classes in the state, which of the following best estimates the number of eighth-grade students in the state who have fewer than two siblings?

Since 34.6 percent of the students had at least two siblings, that means that $65.4 \%$ had less than 2 siblings. Therefore, multiply $.654 \times 26 \times 1800$ to get $30,607.2$.
SO the closest answer is T. 30,600

S 21. The surface area of a cube is $6\left(\frac{a}{4}\right)^{2}$ where $a$ is a positive constant.
Which of the following gives the perimeter of one face of the cube?

There are six faces of a cube so the area of one face is $\left(\frac{a}{4}\right)^{2}$ and each edge must be $\frac{a}{4}$. The perimeter of one face would equal $4 \times \frac{a}{4}=a$.

R_22. The mean score of 8 players in a basketball game was 14.5 points. If the highest individual score is removed, the mean score of the remaining 7 players becomes 12 points. What was the highest score?

The total points of all 8 players $=116$ points. $\quad(8 \times 14.5)$
Without the highest score, the total is 84 points. ( $7 \times 12$ )
So, the highest score was $116-84=32$ points.

C 23. The graph of the linear function $f$ is shown below. The slope of the graph of the linear function $g$ is 4 times the slope of the graph of $f$. If the graph of $g$ passes through the point $(0,-4)$, what is the value of $g(9)$ ?


The slope of $f$ is $1 / 2$. So the slope of $g$ must be $4(1 / 2)=2$.
The graph of $g$ passes through the point ( $0,-4$ ), so its equation must be $g(x)=2 x-4$. Therefore, $g(9)=2(9)-4=14$.

1 24. The equation $x^{2}+20 x+y^{2}+16 y=-20$ defines a circle in the $x y$ plane. What are the coordinates of the center of the circle?

Rewrite the equation to be in the form of the general equation of a circle:
$(x-h)^{2}+(y-k)^{2}=r^{2}$
Complete the squares to get:
$x^{2}+20 x+y^{2}+16 y=-20$
$\left(x^{2}+20 x+100\right)+\left(y^{2}+16 y+64\right)=-20+100+64$
$(x-(-10))^{2}+(y-(-8))^{2}=(12)^{2}$
So, $(h, k)=(-10,-8)$

O 25. Given segment $\overline{P S}, P Q=R S, P Q=x-1, Q R=x, R S=3 x-7$. What is the length of $\overline{P S}$ ?


Set $x-1=3 x-7$
Then $2 x=6$ and $x=3$.
So the length of $\overline{P S}=(3-1)+3+(3(3)-7)=2+3+2=7$.

The droodle used in this puzzle was drawn by Roger Price and appeared in his book called Droodles.

