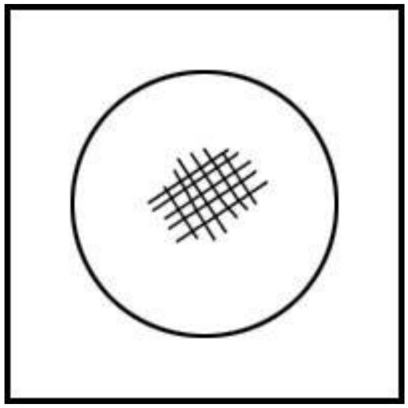
A Droodle for the S.A.T. Math Exam with a calculator

A puzzle by David Pleacher



"A Droodle is a borkley looking sort of drawing that doesn't make any sense until you know the correct title." – Roger Price

Captions for the picture:

#1 M E N D E D   D O U G H N U T .
14 18 5 16 23 16   24 19 11 12 6 20 11 7

#2 S I X   C E N T S   W O R T H   O F
21 15 10   9 18 5 7 21   25 19 1 7 6   19 4

S C R E E N   W I R E .
21 9 1 18 23 20   25 15 1 23

#3 P L A T E   O F   S P A G H E T T I
22 3 17 7 18   19 4   21 22 17 12 6 18 7 7 15

S E R V E D   B Y   N E A T   W A I T E R .
21 23 1 8 18 24   2 13   20 23 17 7   25 17 15 7 18 1

To determine the titles to this droodle, solve the 25 math problems which are similar to problems found on the S.A.T. math section (using a calculator).
Then find the answers to each problem from the choices listed.
Replace each numbered blank with the letter corresponding to the answer for that problem.

R 1. Which expression is equivalent to $(2x^2-4)-(-3x^2+2x-7)$?

R. $5x^2-2x+3$

You need to distribute the minus sign over all three terms in parentheses.

B 2. A website hosting service charges businesses a onetime setup fee of $350 plus *d* dollars for each month. If a business owner paid $1,010 for the first 12 months, including the setup fee, what is the value of *d* ?

The equation $12d+350=1010$ can be used to determine *d*, the number of dollars charged per month for the first 12 months. Subtracting 350 from both sides of this equation yields $12d=660$ and then dividing both sides of the equation by 12 yields $d=55$.

L 3. The table below shows a summary of 1,200 responses to a survey question. Based on the table, how many of those surveyed get most of their medical information from either a doctor or the Internet?

Where Do People Get Most of Their Medical Information?

Source	Percent of those surveyed
Doctor	63%
Internet	13%
Magazines or brochures	9%
Pharmacy	6%
Television	2%
Other or none of the above	7%

According to the table, 63% of survey respondents get most of their medical information from a doctor and 13% get most of their medical information from the Internet. Therefore, 76% of the 1,200 survey respondents get their information from either a doctor or the Internet, and 76% of 1,200 is 912.

_F_ 4. The following table shows the flavors of ice cream and the toppings chosen by the people at a party. Each person chose one flavor of ice cream and one topping. Of the people who chose vanilla ice cream, what fraction chose hot fudge as a topping?

Ice Cream and Topping Selections

		Flavor	
		Vanilla	Chocolate
Topping	Hot fudge	8	6
	Caramel	5	6

According to the table, 13 people chose vanilla ice cream. Of those people, 8 chose hot fudge as a topping. Therefore, of the people who chose vanilla ice cream, the fraction who chose hot fudge as a topping is $\frac{8}{13}$.

_N_ 5. The total area of a coastal city is 92.1 square miles, of which 11.3 square miles is water. If the city had a population of 621,000 people in the year 2010, which of the following is closest to the population density, in people per square mile of land area, of the city at that time?

The land area of the coastal city can be found by subtracting the area of the water from the total area of the coastal city; that is, 92.1 – 11.3 = 80.8 square miles. The population density is the population divided by the land area, or $\frac{621,000}{80.8} = 7,686$, which is closest to 7,690 people per square mile.

_H_ 6. What is the sum of the solutions to $(x-6)(x+0.7)=0$?

The two solutions are $x=6$ and $x=-0.7$, so the sum would equal 5.3

_T_ 7. Given the equation $9ax+9b-6=21$, what is the value of $ax+b$?

To isolate the terms that contain *ax* and *b*, 6 can be added to both sides of the equation, which Gives $9ax+9b=27$. Then, both sides of this equation can be divided by 9, which gives $ax+b=3$

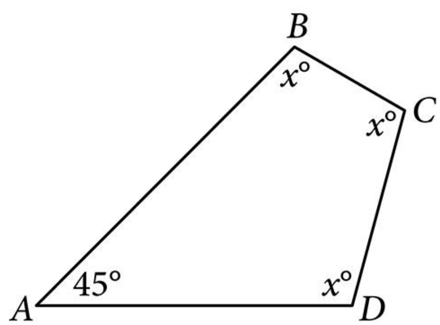
V 8. Kate spent 15% of her 8 hour workday in meetings. How many minutes of her workday did she spend in meetings?

There are 60 minutes in one hour, so an 8 hour workday has 60 times 8, equals 480 minutes.
To calculate 15% of 480, multiply 0.15 by 480: 0.15 times 480 equals 72.
Therefore, Lani spent 72 minutes of her workday in meetings.

C 9. A customer paid $53.00 for a jacket after a 6 percent sales tax was added. What was the price of the jacket before the sales tax was added?

Let x be the price, in dollars, of the jacket before sales tax. The price of the jacket after the 6% sales tax is added was $53. This can be expressed by the equation $x+0.06x=53$ or $1.06x=53$. Dividing each side of this equation by 1.06 gives $x = 50$. Therefore, the price of the jacket before sales tax was $50.

X 10. In the following figure, what is the value of x ?



The figure is a quadrilateral, so the sum of the measures of its interior angles is $360°$. The value of x can be found by using the equation $45+3x=360$. Subtracting 45 from both sides of the equation results in $3x=315$, and dividing both sides of the resulting equation by 3 yields $x=105$. Therefore, the value of x in the figure is 105.

U 11. If 50 one cent coins were stacked on top of each other in a column, the column would be approximately $3\frac{7}{8}$ inches tall.

At this rate, which of the following is closest to the number of one cent coins it would take to make an 8 inch tall column?

A column of 50 stacked one-cent coins is about $3\frac{7}{8}$ inches tall, which is slightly less than 4 inches tall and equivalent to $\frac{31}{8}$. Therefore, a column of stacked one-cent coins that is 4 inches tall would contain slightly more than 50 one-cent coins. It can then be reasoned that because 8 inches is twice 4 inches, a column of stacked one-cent coins that is 8 inches tall would contain slightly more than twice as many coins; that is, slightly more than 100 one-cent coins. An alternate approach is to set up a proportion comparing the column height to the number of one-cent coins, or $\frac{\frac{31}{8}\text{ inches}}{50\text{ coins}} = \frac{8\text{ inches}}{x\text{ coins}}$, where x is the number of coins in an 8-inch-tall column. Multiplying each side of the proportion by 50*x* gives $\frac{31}{8}x = 400$. Solving for x gives $x = 400\left(\frac{8}{31}\right)$, which is approximately 103. Therefore, of the given choices, 100 is closest to the number of one-cent coins it would take to build an 8-inch-tall column.

G 12. If $a-b=12$ and $\frac{b}{2}=10$, what is the value of $a+b$?

Solving the second equation gives you $b = 20$. Substituting that value in the first equation, $a-20=12$, so $a=32$. Therefore, $a + b = 52$.

Y 13. The equation $y=19.99+1.50x$ models the total cost *y*, in dollars, that a company charges a customer to rent a truck for one day and drive the truck *x* miles. The total cost consists of a flat fee plus a charge per mile driven. When the equation is graphed in the x y - plane, what does the *y* - intercept of the graph represent in terms of the model?

The *y*-intercept of the graph of $y=19.99+1.50x$ is the point on the graph with an *x*-coordinate equal to 0. In the model represented by the equation, the *x*-coordinate represents the number of miles a rental truck is driven during a one-day rental, and so the *y*-intercept represents the charge, in dollars, for the rental when the truck is driven 0 miles; that is, the *y*-intercept represents the cost, in dollars, of the flat fee. Since the *y*-intercept of the graph of $y=19.99+1.50x$ is 19.99, the *y*-intercept represents a flat fee of $19.99 in terms of the model.

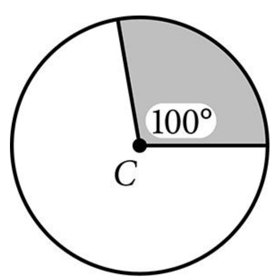
__M__ 14. A cylindrical can containing pieces of fruit is filled to the top with syrup before being sealed. The base of the can has an area of 75 centimeters squared, and the height of the can is 10 centimeters. If 110 centimeters cubed of syrup is needed to fill the can to the top, which of the following is closest to the total volume of the pieces of fruit in the can?

The total volume of the cylindrical can is found by multiplying the area of the base of the can, 75 square centimeters, by the height of the can, 10 centimeters, which yields 750 cubic centimeters. If the syrup needed to fill the can has a volume of 110 cubic centimeters, then the remaining volume for the pieces of fruit is $750-110=640 \text{ cm}^3$.

__I__ 15. The score on a trivia game is obtained by subtracting the number of incorrect answers from twice the number of correct answers. If a player answered 40 questions and obtained a score of 50, how many questions did the player answer correctly?

Let x represent the number of correct answers from the player and y represent the number of incorrect answers from the player. Since the player answered 40 questions in total, the equation $x+y=40$ represents this situation. Also, since the score is found by subtracting the number of incorrect answers from twice the number of correct answers and the player received a score of 50, the equation $2x-y=50$. Adding the two equations, we obtain $3x=30$. Therefore x = 30.

__D__ 16. In the figure below, point C is the center of the circle. What fraction of the area of the circle is the area of the shaded region?



There are 360 degrees in a circle, and it is shown that the central angle of the shaded region is 100 degrees. Therefore, the area of the shaded region can be represented as a fraction of the area of the entire circle, $\frac{100}{360}$, which can be reduced to $\frac{5}{18}$.

A 17. If the ordered pair (x, y) satisfies the system of equations below, what is one possible value of x ?

$$\begin{cases} y = x^2 - 4x + 4 \\ y = 4 - x \end{cases}$$

Set the two equations equal to each other to solve for x.

$x^2 - 4x + 4 = 4 - x$. Then, collecting like terms and factoring:

$x^2 - 3x = 0$

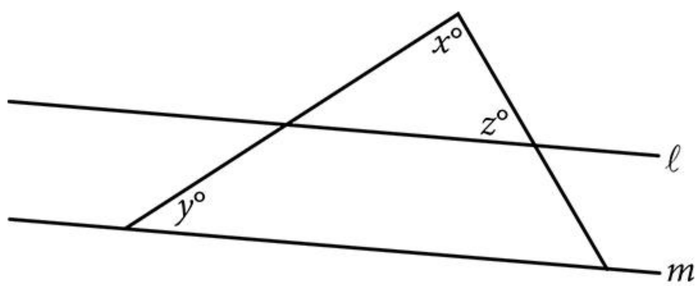
$x(x-3) = 0$

$\therefore x = 0 \text{ or } x = 3$

E 18. In a random sample of 200 cars of a particular model, 3 have a manufacturing defect. At this rate, how many of 10,000 cars of the same model will have a manufacturing defect?

The fraction of the cars in the random sample that have a manufacturing defect is 3 over 200, which equals 0.015. At this rate, out of 10,000 cars there would be 0.015 times 10,000, equals 150 cars that have a manufacturing defect.

O 19. In the figure below, lines l and m are parallel, y equals 20, and z equals 60. What is the value of x ?



Note: Figure not drawn to scale.

The measure of the third angle in the smaller triangle must have the same measure as y because of corresponding angles. Since the three angles of a triangle add up to 180 degrees, $x =$ 100 .

N 20. In the xy-plane, the graph of which of the following equations is a line with a slope of 3 ?

N. $y = 3x + 2$

Use $y = mx + b$, where m stands for the slope.

S 21. In the equation, $x+1=\frac{2}{x+1}$, which of the following is a possible value of $x+1$?

Multiplying both sides of the equation by $x+1$ gives $(x+1)^2=2$.

Taking the square root of both sides yields either $\sqrt{2}$ or $-\sqrt{2}$.

P 22. If $a^{-\frac{1}{2}}=x$, where $a>0$, what is a in terms of x ?

Solving for a, raise each side to the -2 power:

$$\left(a^{-\frac{1}{2}}\right)^{-2}=(x)^{-2}$$

Therefore a = $\frac{1}{x^2}$

E 23. Which of the following is a value of *x* for which the expression $\frac{-3}{x^2+3x-10}$ is undefined?

To be undefined, the denominator must be equal to zero, so set it = 0:

$$x^2+3x-10=0$$

$$(x-2)(x+5)=0$$

$$x=2 \text{ or } x=-5$$

D 24. The table below shows the distribution of ages of the 20 students enrolled in a college class. Which of the following gives the correct order of the mean, median, and mode of the ages?

Ages of 20 Students Enrolled in a College Class

Age	Frequency
18	6
19	5
20	4
21	2
22	1
23	1
30	1

The mode is the data value with the highest frequency. So for the data shown, the mode is 18. The median is the middle data value when the data values are sorted from least to greatest. Since there are 20 ages ordered, the median is the average of the two middle values, the tenth and eleventh, which for these data are both 19. Therefore, the median is 19. The mean is the sum of the data values divided by the number of the data values. So for these data, the mean is

$$\frac{(18\text{x}6)+(19\text{x}5)+(20\text{x}4)+(21\text{x}2)+(22\text{x}1)+(23\text{x}1)+(30\text{x}1)}{20}=20$$

Since the mode is 18, the median is 19, and the mean is 20, mode < median < mean.

W 25. The equation $M=1{,}800(1.02)^t$ models the number of members, *M*, of a gym *t* years after the gym opens. Of the following, which equation models the number of members of the gym *q* quarter years after the gym opens?

In 1 year, there are 4 quarter years, so the number of quarter years, *q*, is 4 times the number of years, *t*; that is, $q=4t$. Solving for *t* and substituting in the equation, we get

$$M=1{,}800(1.02)^{\frac{q}{4}}$$

The droodle used in this puzzle was drawn by Roger Price and appeared in his book called *Droodles.*