The Mathematics of Playing Tic Tac Toe
by David Pleacher

Although it has been shown that no one can ever win at Tic Tac Toe unless a player commits an error, the game still seems to have a universal appeal. While it is true that the number of moves is very large, there are really only a few basic patterns (because of symmetry). In fact, we will see that there are only 12 essentially different games.

   In how many different ways can the first 5 moves be made? _____

$$\underset{1^{st}}{\underline{\quad}} \times \underset{2^{nd}}{\underline{\quad}} \times \underset{3^{rd}}{\underline{\quad}} \times \underset{4^{th}}{\underline{\quad}} \times \underset{5^{th}}{\underline{\quad}} = \underline{\qquad}$$

     But even though there are nine different spaces for your first move, there are essentially only three different places that you could put your opening move (as you will see below).

In the language of Game Theory, Tic Tac Toe is a two-person contest that is finite (comes to an end), has no element of chance, and is played with "perfect information" (all moves being known to both players).

There are three possible opening plays – what are they? _____ , _____, or _____.
Of these three, the strongest is the _____ because the opponent can avoid being trapped at the next move only by one of the 8 possible choices: <u>the center</u>. Conversely, center openings can be blocked only by seizing a _____ . The side opening, in many ways the most interesting because of its richness in traps on both sides, must be met by taking one of 4 cells. Below are the three possible openings. Fill in <u>all</u> the possible moves for the second player in which he could avoid losing.
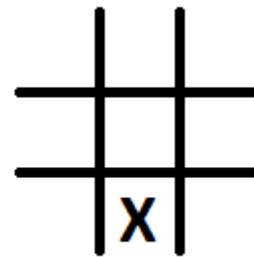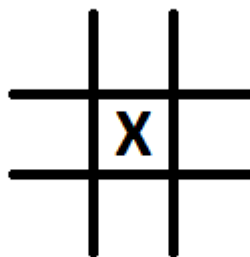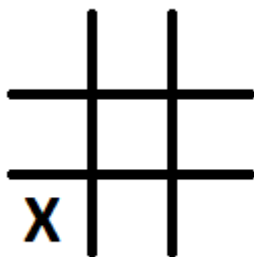
Use the following scheme to answer the questions below:

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1 | 2 | 3
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4 | 5 | 6
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7 | 8 | 9
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Strategies for the side opening:

1. If you open with X8 (X being placed in square #8), and your opponent counters with O2 (O being placed in square #2), what should your move be? _____

_____

2. If your opponent opens with X8, and you play O9, you can trap if she plays X2, X4, or X7.

3. If your opponent opens with X8 and you play O5, and she takes X2, describe your move so that you will win: _____

Variations of Tic Tac Toe:

Some variations of Tic Tac Toe are more interesting to play than the original game. One variation uses 6 counters (one player uses three circles, another uses three triangles). Players take turns placing a counter on the board until all 6 counters are down. If neither player has won by getting 3 in a row, they continue to play by moving on each turn a single counter to any adjacent square (one variation allows only moves horizontally or vertically, while another allows one step in any direction, while still another permits any piece to be moved to any vacant cell). Try some of these.
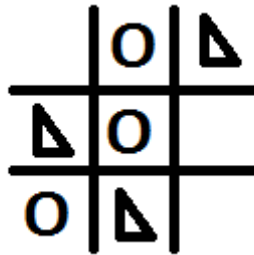
Many variations of moving counter Tic Tac Toe have been applied to 4x4 boards, each player using 4 counters and striving to get 4 in a row. There is an interesting 5x5 version called "Teeko." There are eight markers in a Teeko game, four black and four red. One player, "Black" plays the black markers, and the other, "Red", plays the red. Black moves first, and places one marker on any space on the board. Red then places a marker on any unoccupied space; black does the same; and so on until all eight markers are on the board. The object of the game is for either player to win by having his markers situated in a straight line (vertical, horizontal, or diagonal) or square of four adjacent spaces. If neither player has won after the "drop" (when the eight pieces are put on the board), then it's attempted via the following method: The players alternate moving pieces one at a time, with Black playing first. A piece can only be moved to an adjacent space.

Another variation, called Tac Tic, is played like the original game except that the first player to get 3 in a row LOSES. Explain the strategy of the first player (to insure a draw): _____
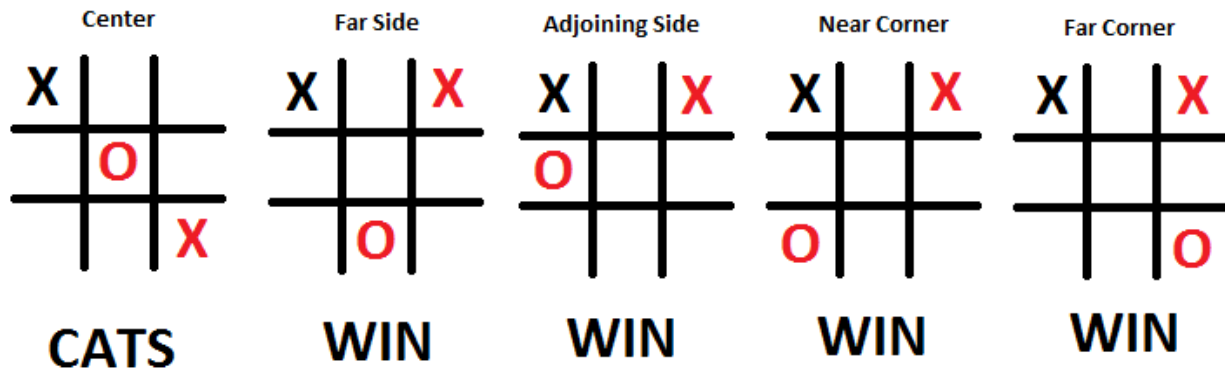_____

The Twelve Essentially Different Tic Tac Toe Games

I. Opening Move #1: The Corner

There are five counter moves for O:

Center, Far Side, Adjoining Side, Near Corner, Far Corner.

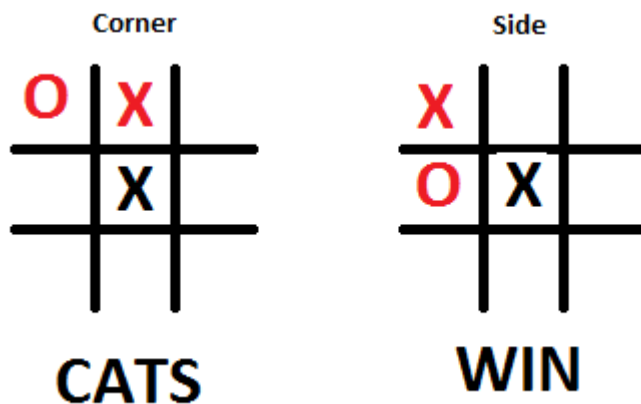All of these will result in a win for X except the Center cell.

Moves in red show the first move for O and the second move for X.

II. Opening Move #2: The Center

There are two counter moves for O:

Corner or Side.

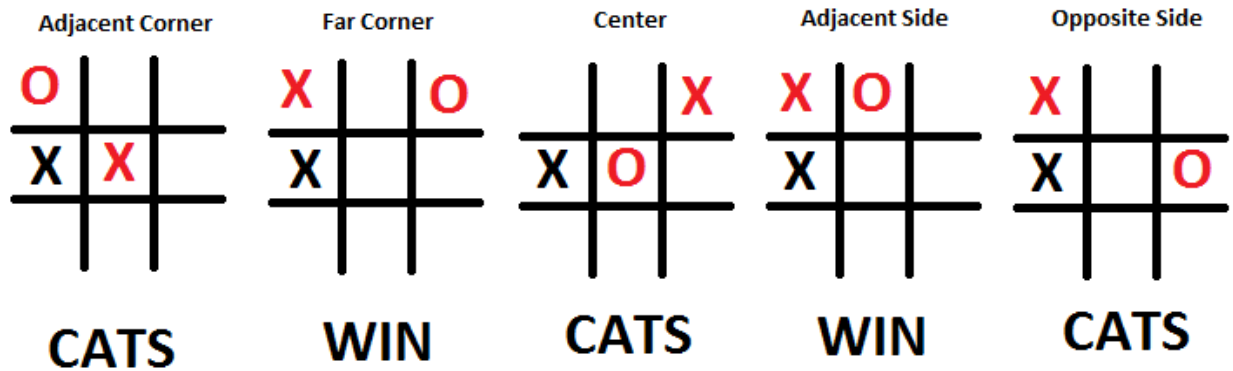The corner results in a CATS game and the side results in a win for X.

III. Opening Move #3: The Side

There are five counter moves for O:

Adjacent Corner, Far Corner, Center, Adjacent Side, Opposite Side.

Three of these result in a CATS game, and two lead to a win for X.

Adjacent Corner	Far Corner	Center	Adjacent Side	Opposite Side
CATS	WIN	CATS	WIN	CATS

Moves in red show the first move for O and the second move for X.