

The unique solution to this problem is 3,816,547,290.

There are  $10!$  (or 3,628,800) combinations of digits in a 10-digit number where each digit is only used once. We can use well-known divisibility rules to drastically reduce the number of possibilities.

1. It is immediately clear that the even digits will be b, d, f, h, and j; and so the odds will be a, c, e, g, and i.
2. Since  $5|abcde$ ,  $e=5$ , and since  $10|abcdefghij$ ,  $j=0$ .
3. Since  $4|abcd$ ,  $4|cd$ . Remembering that c is odd not 5 and d is even not 0, we have that  $cd=12, 16, 32, 36, 72, 76, 92$  or  $96$ . Notice that  $d=2$  or  $6$ .
4. Since  $6|abcdef$ ,  $3|abcdef$ , which means that  $3|(a+b+c+d+e+f)$ . Also,  $3|abc$ , so  $3|(a+b+c)$ . So also  $3|(d+e+f)$ . Remembering that  $d=2$  or  $6$ ,  $e=5$ , and f is even not 0, we have  $def=258$  or  $654$ . Notice that  $f=4$  or  $8$ .
5. Since  $8|abcdefgh$ ,  $8|fgh$ . Remembering that  $f=4$  or  $8$ , g is odd not 5, and h is even not 0, we have  $fgh=416, 432, 472, 496, 816, 832, 872$ , or  $896$ . Notice that  $h=2$  or  $6$ .
6. Now, because we have that both d and h are either 2 or 6, none of the other even numbers can be 2 or 6. Since we already know that  $j=0$ ,  $b=4$  or  $8$ .
7. We already observed that  $3|(a+b+c)$ . Also, a and c are odd not 5, and  $b=4$  or  $8$ , so we have  $abc=147, 183, 189, 381, 387, 741, 783, 789, 981$ , or  $987$ .

From here we can find the solution by “brute force”. Using the numbered items above in the order 7, 4, 5, we fill in the digits from left to right by choosing from the possibilities listed for abc, def, and fgh, making sure that no digits are repeated. Of course j is always 0, and i is chosen as the only digit that remains. We have reduced our possible digit combinations to just ten. Of these possibilities, only one meets all of the divisibility requirements! The others work with everything except the requirement that  $7|abcdefg$ .

a	b	c	d	e	f	g	h	i	j
1	4	7	2	5	8	9	6	3	0
1	8	3	6	5	4	7	2	9	0
1	8	9	6	5	4	3	2	7	0
1	8	9	6	5	4	7	2	3	0
3	8	1	6	5	4	7	2	9	0
7	4	1	2	5	8	9	6	3	0
7	8	9	6	5	4	3	2	1	0
9	8	1	6	5	4	3	2	7	0
9	8	1	6	5	4	7	2	3	0

9	8	7	6	5	4	3	2	1	0
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