Using the Law of Sines and the Law of Cosines to Solve Triangles

by David Pleacher and Carolyn J. Case

1. If *SSS*

Given sides **a, b,** and **c**,   
Use the *Law of Cosines* to determine mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**A**.

* 1. Use the *Law of Cosines* to determine mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B**.
  2. Use the *sum of the angles of a triangle = 180http://www.pleacher.com/mp/mgifs/gifs3/degree.gif* to find mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**C**.

1. If *SAS*
   1. Given sides **a**and **b,** and http://www.pleacher.com/mp/mgifs/gifs3/angle.gif**C**,  
      Use the *Law of Cosines* to determine side **c**.
   2. Use the *Law of Cosines* to determine http://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B**.
   3. Use the *sum of the angles of a triangle = 180http://www.pleacher.com/mp/mgifs/gifs3/degree.gif*to find mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**A**.
2. If *ASA*
   1. Given mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**A** and mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B** and side **c**,   
      Use the *sum of the angles of a triangle = 180http://www.pleacher.com/mp/mgifs/gifs3/degree.gif* to find mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**C**.
   2. Use the *Law of Sines* to determine side **b**.
   3. Use the *Law of Sines* to determine side **a**.
3. If *AAS*
   1. Given mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**A** and mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B** and side **a**,   
      Use the *sum of the angles of a triangle = 180http://www.pleacher.com/mp/mgifs/gifs3/degree.gif* to find mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**C**.
   2. Use the *Law of Sines* to determine side **b**.
   3. Use the *Law of Sines* to determine side **c**.
4. If *SSA* (Ambiguous Case)
   1. Given sides **a**and **b,** and http://www.pleacher.com/mp/mgifs/gifs3/angle.gif**A**,  
      Use the *Law of Sines* to solve for sin http://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B**.
      1. If sinhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B** > 1,   
         There is **no** triangle.
      2. If sinhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B** http://www.pleacher.com/mp/mgifs/gifs3/lte.gif 1,   
         Determine mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B** in quadrant I.
         1. If mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**A** + mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B** http://www.pleacher.com/mp/mgifs/gifs3/gte.gif 180http://www.pleacher.com/mp/mgifs/gifs3/degree.gif   
            There is **no** triangle.
         2. If mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**A** + mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B** < 180http://www.pleacher.com/mp/mgifs/gifs3/degree.gif   
            There is at least one triangle.
            1. Determine mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B** in quadrant II.   
               It has the same sine value as http://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B** .  
               Call this angle, http://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B'**.
            2. Determine mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**A** + mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B'**

If mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**A** + mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B'** http://www.pleacher.com/mp/mgifs/gifs3/gte.gif 180http://www.pleacher.com/mp/mgifs/gifs3/degree.gif   
There is only one triangle.

Determine mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**C** using the *sum of the angles in a triangle = 180*http://www.pleacher.com/mp/mgifs/gifs3/degree.gif

Determine side **c** using the *Law of Sines*.

If mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**A** + mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**B'** < 180http://www.pleacher.com/mp/mgifs/gifs3/degree.gif   
There are two triangles.

Determine mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**C** using the *sum of the angles in a triangle = 180*http://www.pleacher.com/mp/mgifs/gifs3/degree.gif

Determine side **c** using the *Law of Sines*.

Determine mhttp://www.pleacher.com/mp/mgifs/gifs3/angle.gif**C'** using the *sum of the angles in a triangle = 180*http://www.pleacher.com/mp/mgifs/gifs3/degree.gif

Determine side **c'** using the *Law of Sines*.