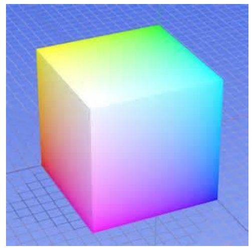
Graph Theory and the Konigsberg Bridge Problem

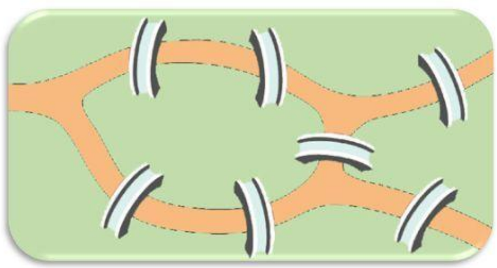
Answer Key by David Pleacher

WHO IS THIS FAMOUS MATHEMATICIAN?

1. He is credited with discovering this formula: VERTICES + FACES = E DGES + 2



2. P u zzles like the seven bridges of Konigsberg interested him and were part of a new branch of mathematics that he started called topology.



Can you find a path through the city that would cross each bridge once and only once?

3. He had thirteen grandchildren and died while playing with a grandchild. He is said to have created mathematics with a baby on his l ap and children playing around him.

4. He contributed greatly to the foundations of every branch of advanced mathematics. One of his famous equations is: $e = 1 + \frac{1}{1} + \frac{1}{1 \bullet 2} + \frac{1}{1 \bullet 2 \bullet 3} + \frac{1}{1 \bullet 2 \bullet 3 \bullet 4} + \ldots$

5. So numerous were his mathematical manuscripts that 200 volumes will be required to organize them into book form. He wrote 800 pages a year of high quality mathematics.

6. His name is Leonard E U L E R .

   1 2 3 4 5

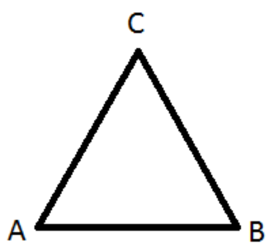
For each of the following diagrams, fill in the chart on the next page and

(1) Tell how many odd and even vertices the figure has and

(2) Determine whether you can draw each design without retracing a line or lifting the pencil. If the figure can be redrawn, give the starting and ending points.

Odd vertices are those points which have an odd number of lines / curve segments connecting to it.

Even vertices are those points which have an even number of lines / curve segments connecting to it.



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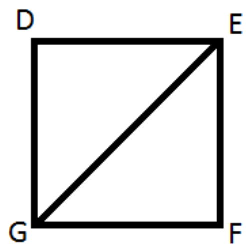

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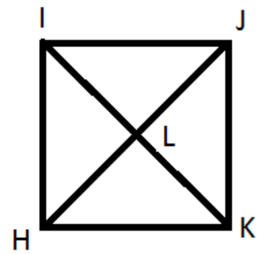

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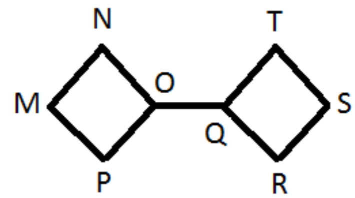

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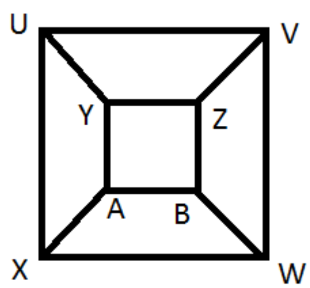

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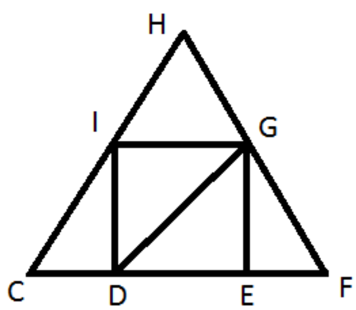

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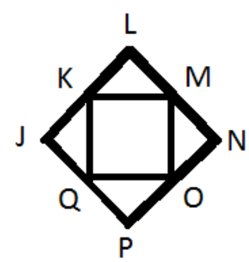

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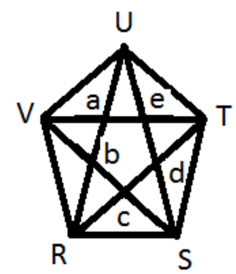

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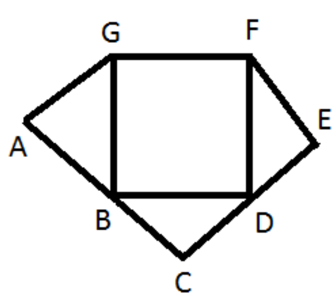

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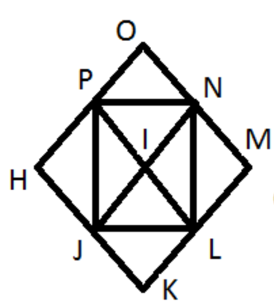

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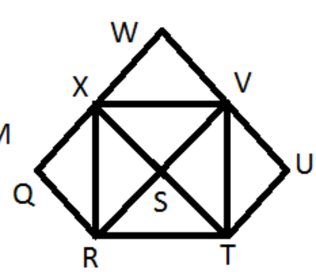

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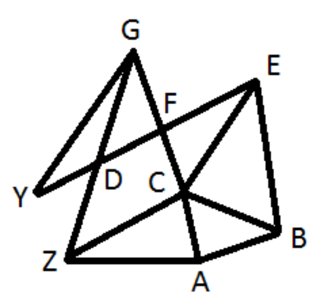
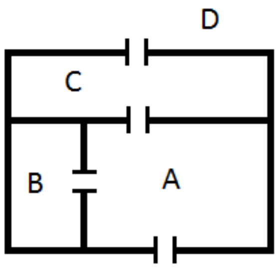


#12

Can you go through each door in the following house plans exactly once?
Consider each room as a vertex and the outside as one vertex.



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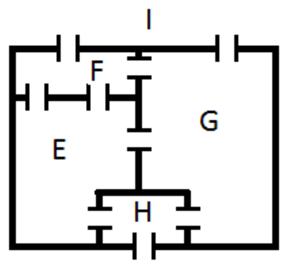

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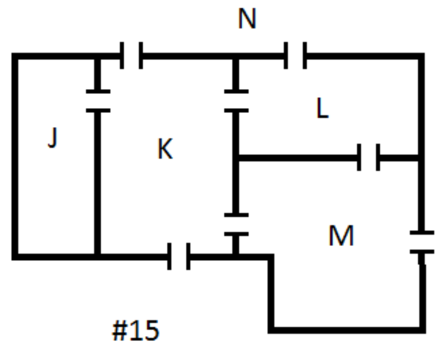


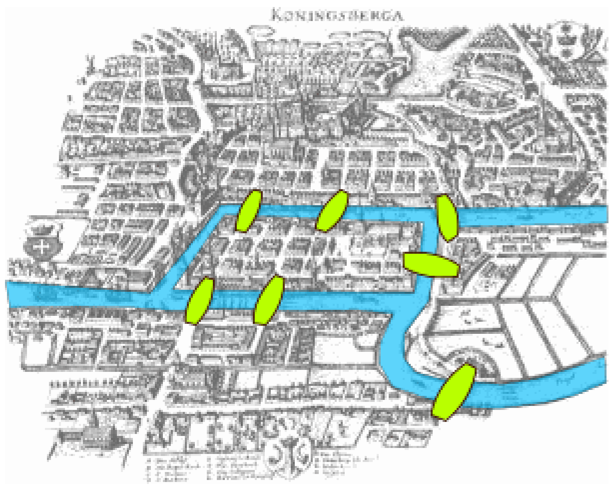
#15

Figure #	Number of Even Vertices	Number of Odd Vertices	Can it Be Drawn? Start? End?
1	3	0	Yes, anywhere
2	2	2	Yes, G, E
3	1	4	no
4	6	2	Yes, O, Q
5	0	8	no
6	5	2	Yes, G, E
7	8	0	Yes, anywhere
8	10	0	Yes, anywhere
9	5	2	Yes, F, G
10	5	4	no
11	6	2	Yes, X, V
12	3	6	no
13	2	2	Yes, A, B
14	3	2	Yes, H, I
15	1	4	no

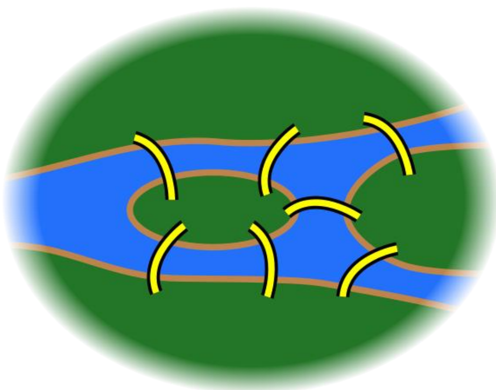
What is the common characteristic of the figures that can be drawn?

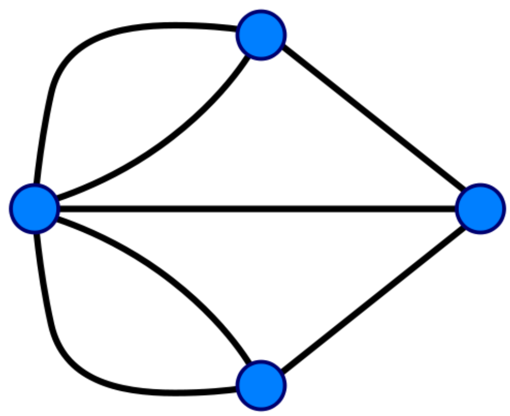
Only figures with 0 or 2 odd vertices can be redrawn without lifting your pencil or retracing a path.
It is impossible to draw a figure with an odd number of odd vertices (it must have 0, 2, 4, 6, …)!
If you have zero odd vertices, then you can begin at any point, and you will end at that point.
If you have 2 odd vertices, then you must begin at one and end at the other.
If you have 4 (or more) odd vertices, you would start at one, but you can't end up at 3 different points!

Now apply what you have learned to the Konigsberg Bridge Problem that Leonard Euler solved.
Here is a map of Konigsberg, Prussia, which is now called Kaliningrad, Russia.



First, identify the two islands and the two main banks of the city and the river Pregel and the 7 bridges.
Then replace each land mass by a vertex and each bridge by a line / curve segment.





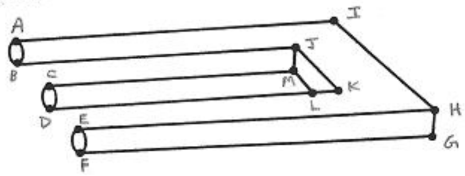
This network is similar to the diagrams #1 – 12 above.

How many EVEN vertices does the figure have? <u>0</u>

How many ODD vertices does the figure have” <u>4</u>

Can it be redrawn? <u>No, because it has more than 2 odd vertices.</u>

If we think of the peculiar figure on the cover of *Mad* magazine as a network instead of solid, it will look like the figure below.



This network is similar to the diagrams #1 – 12 above.

How many EVEN vertices does the figure have? 3

How many ODD vertices does the figure have” 10

Can it be redrawn? No, because it has more than 2 odd vertices.